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An Experimental Study of Floc Strength

Photographic observation of the disintegration of individual flocs upon interaction with a turbulent jet has been used to quantify the binding force of kaolin-polymer and kaolin-Fe^{+III} aggregates. The data suggest that the strength per unit mass decreases with increasing floc size, thereby supporting the multiple-level model for aggregation.

L. A. GLASGOW and JYH PING HSU
Department of Chemical Engineering
Kansas State University
Manhattan, KS 66506

SCOPE

An important characteristic of flocs in coagulation processes is their binding force, i.e., the ability of the aggregate structure to resist deaggregation. The impact of the physicochemical environment during coagulation upon floc strength is perhaps qualitatively understood, but present knowledge is not adequate to permit rational process improvement. The objectives of this investigation were to observe the disintegration of individual flocs photographically and to deduce, if possible, the magnitude of the force required to produce breakup. Different floc structures were studied at various levels of acidity to produce quantitative information about the effects of these parameters upon aggregate resistance.

Previous studies of floc strength have relied upon indirect measurements to deduce relative binding force. Hannah et al. (1967) measured the size distribution of fragments produced when a parent aggregate was drawn through a 70 μm orifice; they manipulated mixing intensity, pH, and coagulant type. Tambo and Hozumi (1979) observed maximum floc diameter in a paddle flocculator as a function of pH, dissipation rate, and coagulant type. Either of these cited investigations can serve as the basis for valuable generalizations, but neither can be used to make a quantitative estimate of floc strength.

CONCLUSIONS AND SIGNIFICANCE

Photographic observation of the breakup of individual flocs in a turbulent jet has been used to quantify the binding force of aggregates as a function of the flocculator environment during coagulation. Flocs formed from a dispersed phase of kaolin with an anionic macromolecular coagulant-aid were found, at a given size, to be more than twice as strong as those formed with an inorganic acid salt ($FeCl_3$) alone. This result is interpreted as a consequence of structural differences, with interparticle bridging in the case of the clay-polymer structures producing mechanically-linked flocs that are quite resilient when subjected to minor deformation. Floc growth in the case of the Fe^{+III} coagulant occurs under alkaline conditions by particle capture by enmeshment; structures thus produced are relatively weak. The variation of binding force with pH was investigated for both types of coagulants and increased alkalinity produced stronger floc in both systems. This effect was more pronounced in the case of kaolin-Fe^{+III} structures. It was found that floc binding force in all cases examined could be correlated with floc size in a manner consistent with the spectral model of Tambo and Hozumi (1979), even though their model is not a physically realistic portrait of the breakup process in conventional coagulation environments.

Increased floc strength could be used in a coagulation process to produce larger, more settlable aggregates. Floc growth rates could also be enhanced since stronger structures are better able to tolerate fluid velocity gradients required for orthokinetic flocculation. It is likely, however, that stronger flocs would be more difficult to dewater.

The information contained in this paper contributes to the understanding of deaggregation and can be used for the refinement of floc breakage models that contain binding force as a parameter; indeed, Glasgow and Luecke (1980) have demonstrated the pressing need for this type of investigation. The photographic observations of the breakup of individual aggregates in a turbulent environment appear to be the first of their kind in the literature. In addition, the data obtained from them should facilitate optimization of coagulation processes where particular objectives or constraints must be satisfied.

INTRODUCTION

It is evident that any model of floc breakage will require floc binding force, or strength, as a parameter. Thomas (1963) has used an expression developed by Hamaker (1937) to roughly estimate the force of attraction between colloidal particles

$$F = \frac{Ar}{12h^2} \tag{1}$$

By representing the number of particle-particle contacts per unit volume floc as n_c, Thomas was able to write an expression for the

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Correspondence concerning this paper should be addressed to L. A. Glasgow.

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floc yield stress

$$\tau_y = aFn_c{}^d \tag{2}$$

This model is supposed to be based upon replacement of the London-Van der Waals attraction by an infinitely deep potential well, even though particle contact in such a case would be irreversible. A problem as pressing as this error in logic, however, is that practical coagulation usually involves significant coulombic repulsion; particle contact and even close approach may be nonexistent in typical floc structures. In this case, a macromolecular coagulant-aid that functions by adsorption and interparticle bridging may be able to span the separation maintained by repulsive forces.

Healy and La Mer (1964) have proposed a rate constant in a model of the redispersion of flocs formed with polyacrylamide coagulant-aids that depends upon surface coverage, θ. They express the rate of liberation of primary particles as

$$\frac{dn_1}{dt} = \frac{k_2}{\theta(1-\theta)}r \tag{3}$$

They suggest that the strength of the flocs may be determined by the product $\theta(1-\theta)$, which has a maximum value at $\theta = 0.5$, i.e., half surface coverage by adsorbed polymer segments.

Tambo and Hozumi (1979) have suggested that the floc binding force is equal to the product of a strength constant, σ, and the net sectional area at the breakage plane

$$B = \sigma A_f \tag{4}$$

By expressing the net sectional area in terms of floc diameter and porosity, Eq. 4 can be written

$$B \sim d^2(1-p)^{2/3}\sigma \tag{5}$$

Presumably σ would be assigned different values for different floc structures.

It is apparent that once the floc yield stress or binding force enters the purview of a floc breakup model, a logical step is to relate it to hydrodynamic forces. Parker et al. (1972), Delichatsios and Probstein (1975), and Tambo and Hozumi (1979) have used an argument presented by Levich (1962) in his description of droplet breakup in turbulent flow. Through use of the familiar $-5/3$ power law for the inertial subrange of the spectrum of turbulent energy, the magnitude of the relative velocity on opposite sides of the floc can be represented by

$$u \sim (\epsilon d)^{1/3} \tag{6}$$

Since the dynamic force experienced by the floc per unit area is proportional to ρu^2, the total force required for breakup is the product of the dynamic head and the floc surface area

$$B \sim \rho\epsilon^{2/3}d^{8/3} \tag{7}$$

A condition for maximum stable floc size can clearly be established by equating Eq. 7 with an expression for the floc binding force such as Eq. 5, or Eq. 2, if first multiplied by the floc surface area. It must be emphasized, however, that Eq. 7 requires an integral-scale Reynolds number large enough ($\gtrsim 10^4$) to produce an inertial subrange in the spectrum of turbulent energy; this condition is not likely to be realized in typical coagulation processes.

FLOC STRENGTH MODEL

In cases of "rapid" coagulation where coulombic repulsion is suppressed by the addition of electrolyte and the Van der Waals-London forces of attraction are hypothetically replaced by an infinitely deep potential well, particle-particle encounter produces permanent contact. A more realistic view of colloidal particle interaction could be based upon the sum of the repulsive and attractive potentials, with the force of interaction given by

$$F = -\left(\frac{\partial\psi}{\partial h}\right) \tag{8}$$

However, as Overbeek (1952) has noted, interparticle repulsion has a range on the order of the thickness of the double layer. Van Olphen (1963) has suggested that the Debye length, δ, can be used to estimate this thickness for the conditions of the investigation

$$\delta = \sqrt{\beta \mathrm{kT}/(4\pi \mathrm{e}^2\Sigma \mathrm{N_i z_i}^2)} \tag{9}$$

Typical values of δ for our experiments were on the order of 100 Å, yet the overall average interparticle distances in the larger flocs were estimated from limited density data to be on the order of 10^4 Å. We infer, therefore, that the interaction potential is not significant in the determination of floc strength in the kaolin-anionic polymer case. V. K. La Mer has voiced the same opinion; see the discussion following Healy and La Mer (1962).

Indeed, in many practical applications, floc binding force or strength will be determined by the coagulating agent and the level of aggregation at which the particular aggregate was formed. Michaels and Bolger (1962) and Vold (1963) have advocated multiple level floc growth; Vold proposed three levels, consisting of: primary particles, small compact flocs whose growth is characterized by the addition of primary particles, and large, loose affiliations of flocs of the second level. Under the proper hydrodynamic conditions, it is not difficult to imagine additional levels of increasing complexity. It is our present purpose to suggest a rational model for description of floc strength as a function of size.

We begin by postulating a multiple-level growth mechanism that qualitatively explains the relationship between buoyant density and size. Since upper-level aggregates exhibit lower densities than their more compact antecedents, they can also be expected to be less resistant (per unit mass or volume) to disruptive forces. The density of second-level aggregates formed from clay dispersions by anionic polyelectrolytes is determined by the conformation of the adsorbed macromolecule, with low pH and ionic strength producing coiled, spherical shapes and high pH and ionic strength promoting extended, rod-like conformations. The strength of this type of low-level aggregate is determined directly by the mean number of interparticle macromolecular bridges formed between adjacent primary particles. As flocculation time increases, fewer and fewer polymer segments extend far enough from the clay surface to perform a bridging function. Therefore, the macromolecular network of upper-level structures is far less extensive, leading to reduced strength per unit volume.

We begin the model development by assuming that each primary particle added to a second-level structure brings with it a set fraction of immobilized water. If n_1 is the total number of primary particles in a second-level aggregate and v_1 their individual volume,

$$v_2 = (1+\alpha)n_1 v_1 \tag{10}$$

The density of such structures, therefore, can be written as

$$\rho_2 = \frac{1}{1+\alpha}[\rho_1 + \alpha\rho_f] \tag{11}$$

If the fraction of immobilized water is assumed constant, subsequent-level structures can be treated analogously, yielding

$$v_{j+1} = (1+\alpha)n_j v_j \tag{12}$$

and

$$\rho_{j+1} = \frac{1}{1+\alpha}[\rho_j + \alpha\rho_f] \tag{13}$$

The essential characteristics of this model are illustrated in Figure 1, of course, the delineation between various levels is not sharply defined due to the random nature of the aggregation progress. Furthermore, the variation of density with size will depend upon the mean number of component structures at a given level, i.e.,

$$d_{j+1} = \left[\frac{6}{\pi}(1+\alpha)n_j v_j\right]^{1/3} \tag{14}$$

In the case of kaolin-polymer floc strength, however, the dependence upon size will occur as a result of the interplay between floc growth and polymer adsorption kinetics. We now demonstrate

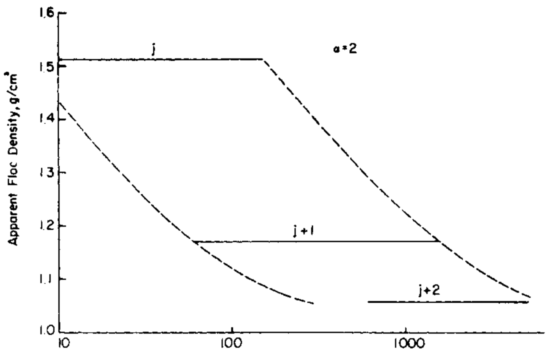

Figure 1. Multiple-level model floc density from Eqs. 12 and 13.

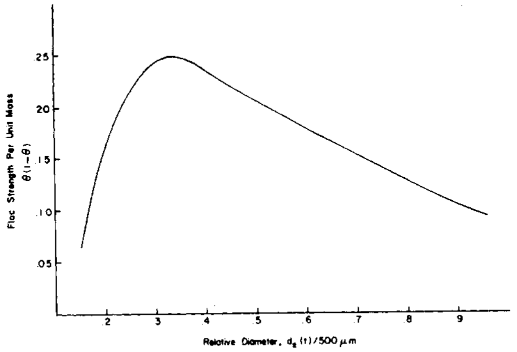

Figure 2. Relative floc strength of second-level aggregates from Eqs. 16 and 21.

development of an approximate description of floc strength during coagulation; the model is restricted to small values of t so that the system can be treated as through it were monodisperse. Peterson and Kwei (1961) have successfully fit the Langmuirian kinetic formula describing fractional surface coverage

$$\frac{d\theta}{dt} = k_1(1-\theta)c - k_1'\theta \qquad (15)$$

to experimental data for the adsorption of polyvinyl acetate upon chrome plate. We modify Eq. 15 by adopting Jankovics (1965) assumed relation between polymer concentration, c, and surface coverage, θ: $c = (c_o - c_1\theta)$, and by assuming desorption to be negligibly small. Equation 15 can be integrated with the boundary condition $\theta = 0$ at $t = 0$ to yield

$$\frac{1}{c_o - c_1} \, ln \left(\frac{c_o - c_1\theta}{c_o(1-\theta)}\right) = k_1 t \qquad (16)$$

We shall assume that clay-polymer floc strength per unit mass is proportional to $\theta(1 - \theta)$ as suggested by Healy and La Mer (1964).

The rate of disappearance of primary particles by orthokinetic flocculation can be rendered first order with respect to n_1 through use of the volume fraction of the dispersed phase, Ω (Weber, 1972)

$$\frac{dn_1}{dt} = -\frac{4}{\pi}\eta G\Omega n_1 \qquad (17)$$

where $n_1 = n_{1_o}$ at $t = 0$.

In Eq. 17, η, a collisional efficiency factor, has been introduced to account for the increase in colloid particle collision radius resulting from polymer adsorption.

According to the multiple-level aggregation model, Equation (10)

$$\frac{dV_2}{dt} \sim -(1+\alpha)v_1\frac{dn_1}{dt} \qquad (18)$$

where $V_2 = 0$ at $t = 0$.

By integration of Eq. 17 and substitution into the integrated form of Eq. 18

$$V_2 \sim (1+\alpha)v_1 n_{1_o}(1 - e^{-\lambda t}) \qquad (19)$$

where $\lambda = 4/\pi \; \eta G \Omega$ and V_2 is the total volume formed at the second level of aggregation per unit volume. It is reasonable to assume that the total number of second-level structures undergoes a rapid initial increase until parity is achieved between the rate of formation from level-one particles and the rate of depletion (due to third-level floc formation). For small t's, we propose

$$\frac{dn_2}{dt} = \frac{\lambda n_1}{b} - k_3 n_2^2 \qquad (20)$$

as a suitable approximation. The seed parameter, b, has been introduced to account for the fact that large numbers of primary particles affiliate with a single second-level nucleus. b can be estimated with an experimental value for $\left.\frac{dn_2}{dt}\right|_{t=0}$ since λ and n_{1_o} are known and $n_{2_o} = 0$. Therefore,

$$v_2 \simeq \frac{(1+\alpha)v_1 n_{1_o}(1 - e^{-\gamma t})}{n_2(t)} \qquad (21)$$

The initial variation of floc strength with size will be determined by the interplay between the product $\theta(1 - \theta)$ as calculated with Eq. 16 and the second-level floc size as given by Eq. 21. Figure 2 illustrates the type of behavior expected; approximate parameter values were determined by aggregation experiment.

EXPERIMENTAL DETAILS

Sample flocs were formed in conventional jar-test apparatus by the aggregation of colloidal kaolin with either $FeCl_3$ or a high molecular weight polyacrylamide at a constant ionic strength of 0.003 M and pH's ranging from 7.2 to 10.6. In the case of the polymer coagulant, optimal treatment dosage was determined by turbidity reduction in jar tests. Individual aggregates were transferred from the beakers by serological pipet to a polycarbonate observation chamber shown in Figure 3 and allowed to settle. A horizontally-directed two-dimensional turbulent jet issued forth into the observation chamber at a level 0.32 m below the point of floc introduction. The resulting displacement and fragmentation of the test aggregate was observed photographically using a single-frame, multiple flash technique; the camera used was a conventional 35 mm single-lens reflex with a 55 mm, f 2.8 macro-lens. Kodak Plus-X Pan film in 36-exposure cassettes were used to record particle behavior; the film was processed at ASA 400.

The jet entered the apparatus through a rectangular slot, 0.1 m long and 0.003 m high. During the course of the experimental work, jet Reynolds numbers ranging from about 1,100 to 5,400 were used. Semi-quantitative information regarding the flow characteristics was obtained from photographic flow visualization by seeding the liquid phase with neutrally-buoyant glass microballons. Relative axial intensities were measured on the jet center-line at distances ranging from 0.015 m to 0.333 m downstream from the slot opening; $\sqrt{\overline{u_1^2}}/U_1$ was about 30 percent at $x_1 = 0.015$ m when Re = 3740. The longitudinal correlation coefficient $(\overline{R_{11}(r_1)})$ was also measured along the jet centerline; the integral length scale obtained by integration of R_{11} was 0.0067 m.

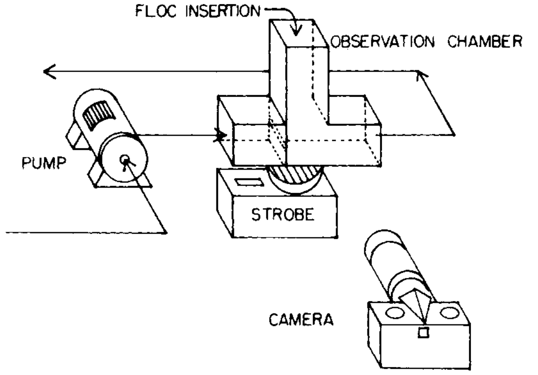

Figure 3. Experimental apparatus for study of individual floc breakup.

The strobe interval used for kaolin-Fe^{+III} flocs was 100 ms; 50 ms was used for kaolin-polyacrylamide structures. Sample photographs are shown in Figures 4, 5, 7, 8 and 9. Approximately 800 such photographs were taken during the course of the investigation, however, due to the very shallow depth of field of the macro lens only about 150 provided usable information. The magnification factor obtained in the enlargements was 3.65. Each usable photograph was examined and measured manually to obtain the horizontal and vertical distances between successive images; knowledge of the degree of magnification and the strobe interval would then permit computation of the particle velocity as a function of time. Therefore, the critical acceleration producing breakup could be deduced from the slope of the velocity-time curve immediately prior to the image of disintegration. Additionally, the photographic evidence could be used to classify the breakup process as erosive, thorough, or binary, according to the forms suggested by Narsimhan et al. (1980) in their discussion of droplet breakage.

Particle-size distribution measurements were made from the photographic record using a Bausch & Lomb Omnicon Image Analysis System with macroviewer. The advantages of this technique are manifest: the testing is non-destructive and no ionic strength alterations are necessary. The minimum detectable particle size is determined by the resolution of the lens-film combination; typically this value is approximately 10×10^{-6} m. Sizing and counting can be performed very rapidly, with a savings of many man-hours over previous techniques.

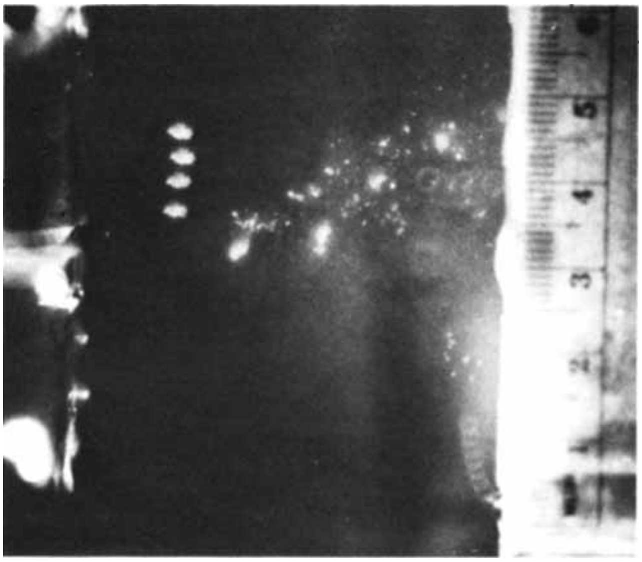

Figure 5. Breakage of kaolin-Fe^{+III} floc at Re-1386 and $x_1 \simeq 0.01$ m.

EXPERIMENTAL RESULTS AND DISCUSSION

Although our primary purpose was to study the strength of kaolin-polyacrylamide aggregates, kaolin-Fe^{+III} flocs were also included so that the effects of pronounced structural differences could be noted. The kaolin-$Fe(OH)_3$ flocs selected for the deaggregation experiments ranged in size from about 700×10^{-6} m to 3500×10^{-6} m and the photographic record clearly shows that the breakup mechanism entails massive splitting; primary particle erosion did not contribute significantly to parent particle size reduction. Since the strobe interval in the case of kaolin-Fe^{+III} flocs was 100 msec, periodic disturbances with frequencies much greater than about 1 Hz could not be detected. This is of no real importance, however, as the observed breakup could only be caused by stresses improved by the mean flow or by integral-scale eddies during the entrainment process. Figure 4 illustrates the thorough breakage of a large kaolin-iron floc at a jet Reynolds number of 1386; the mean fragment size is about 275×10^{-6} m. The geometric mean parent aggregate diameter was 2680×10^{-6} m. A far less severe case is shown in Figure 5; although the jet Reynolds number is again 1,386, the floc encountered the edge of the jet at $x_1 \simeq 0.01$ m instead of the $x_1 \simeq 0.005$ m distance shown in Figure 4. The breakage incident illustrated by Figure 5 could perhaps be termed "erosive" since the parent aggregate is reduced to one very large fragment and several daughter particles of sizes comparable to those found in the case of thorough breakage. The interpretation of critical acceleration or force from such photographs is complicated by the variation in severity of the breakage event due to the distance of initial floc-jet interaction from the jet origin ($x_1 = 0$). Nevertheless, it was found that the critical force requirement for kaolin-$Fe(OH)_3$ flocs could be correlated with aggregate size. The photographic record in each case was enlarged and the displacement between successive images was measured. Since the strobe interval was known, the inertial force acting upon the floc could be estimated, with the largest value preceding disintegration being taken as the critical force. For large flocs formed at an ionic strength of 0.003 M and a pH of 9.5, regression analysis of 52 data points yielded the relationship

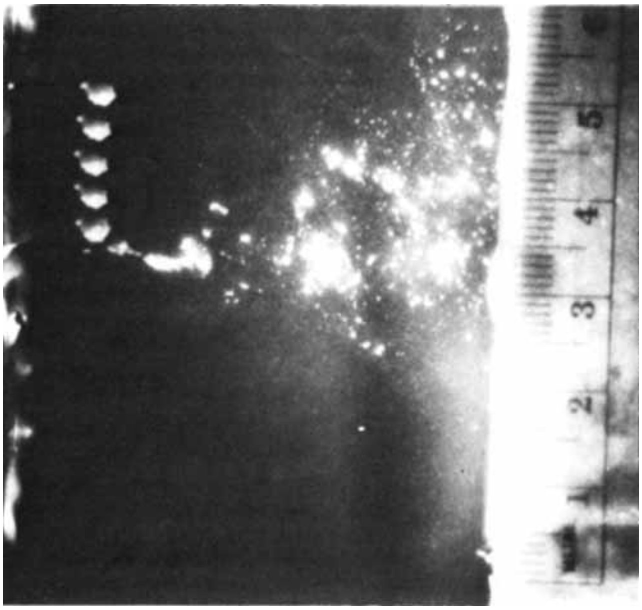

Figure 4. Breakage of kaolin-Fe^{+III} floc at Re = 1386 and $x_1 \simeq 0.005$ m.

$$B = 13.56d^{2.40} \tag{22}$$

with a correlation coefficient of 0.94. A similar analysis for kaolin-Fe^{+III} flocs formed at a pH of 7.4 gave

$$B = 8.54d^{2.40} \tag{23}$$

where, as in the case of Eq. 22 the critical force, B, is in dynes, and the parent aggregate diameter, d, in cm. The difference in the

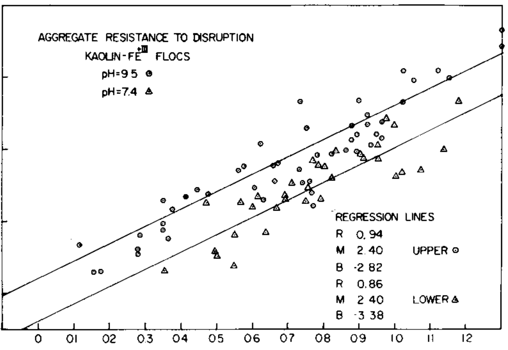

Figure 6. Experimental strength measurements for kaolin-Fe^{+III} flocs.

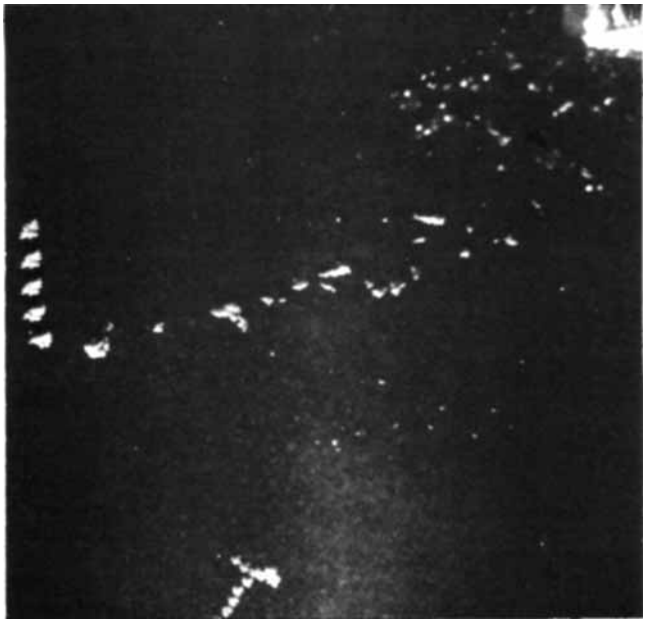

Figure 7. Breakage of kaolin-polyacrylamide floc at Re = 3780 and $x_1 \simeq 0.013$ m.

leading coefficient between Eqs. 22 and 23 is seen as a consequence of the more complete hydrolysis of the trivalent iron under more alkaline conditions. We presume that floc growth in the kaolin-Fe^{+III} system at the second level of aggregation takes place by particle capture by enmeshment in the hydroxide precipitate. According to Stumm and Morgan (1970), the alkalinity levels used in our experiments should produce either the uncharged species, $Fe(OH)_3 \cdot (H_2O)_3$, or anionic hydroxo complexes such as $[Fe(OH)_4 \cdot 2H_2O]^-$; however the production of isopolycations in intermediate steps of the hydrolysis may be significant at pH = 7.4. The experimental strength measurements for kaolin-Fe^{+III} flocs are summarized in Figure 6.

In the case of flocs formed with a high molecular weight polyacrylamide, aggregation at the second level takes place by interparticle bridging. Thus, mechanical linkages exist between adjacent primary particles and flocs of this type can be expected to be somewhat more resistant to physical deformation. Figure 7 illustrates the interaction of a 2270×10^{-6} m kaolin-polyacrylamide aggregate with the turbulent jet (Re = 3780) where the initial value of x_1 is about 0.013 m. The resulting disintegration might be characterized as thorough; although the number of daughter particles produced is relatively small, their size is nearly uniform with a mean diameter of about 980×10^{-6} m. Figure 8 shows the effects of the jet (Re = 1903) upon a very large aggregate (3530×10^{-6} m) with the initial interaction occurring at $x_1 \simeq 0.03$ m. The resulting breakage is nearly binary with major fragments produced at $d \simeq 2600 \times 10^{-6}$ m and $d \simeq 1770 \times 10^{-6}$ m. Figure 9 is an example of very thorough breakage with Re = 2646 and the initial encounter at $x_1 \simeq 0.005$ m. In this case, the mean daughter particle size is 375×10^{-6} m. In these three examples of kaolin-polyacrylamide floc breakup, the parent structures were very large and certainly formed at an upper level of aggregation. We suggest that the mean size of fragments produced under very severe conditions, (for example, Figure 9) corresponds roughly to second-level structures of maximum strength. Therefore, it should be possible (with the addition of an appropriately designed coagulation experiment) to quantify the seed and rate parameters in Eq. 20.

In order to obtain estimates for the parameters in Eqs. 16 and 20, a coagulation experiment was conducted in which kaolin-

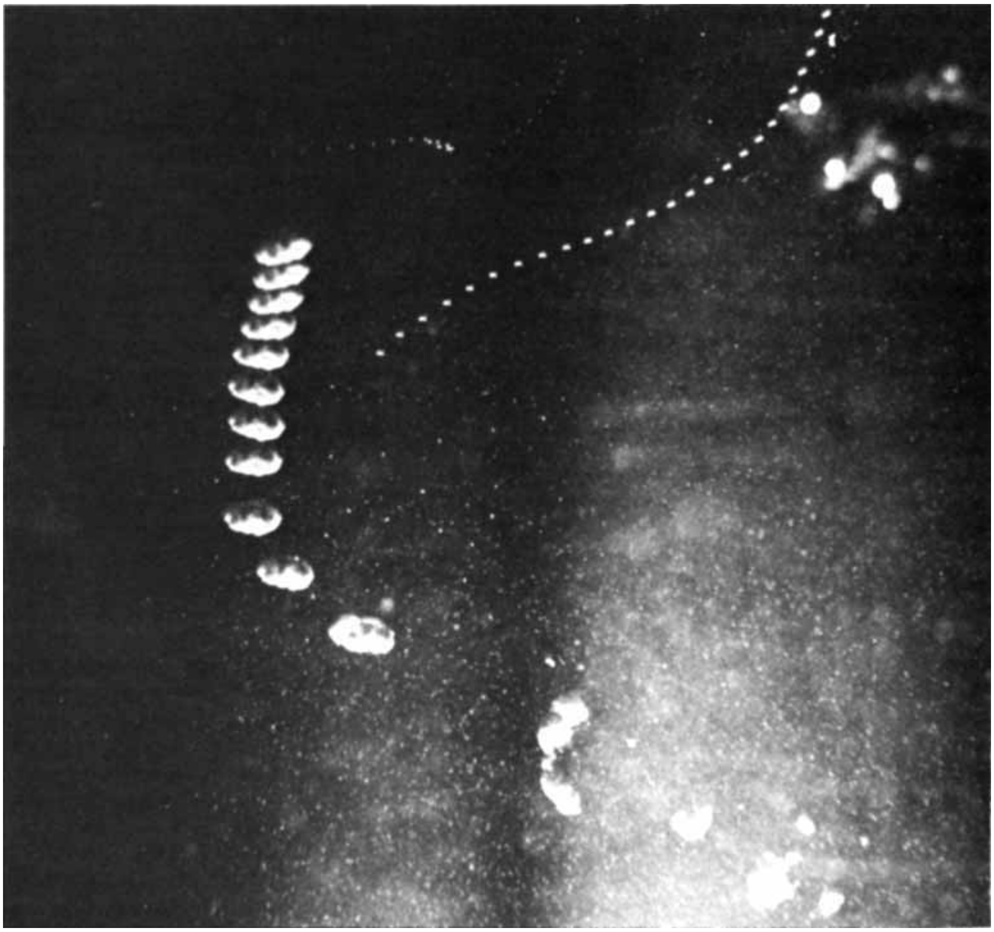

Figure 8. Breakage of kaolin-polyacrylamide floc at Re = 1903 and $x_1 \simeq 0.03$ m.

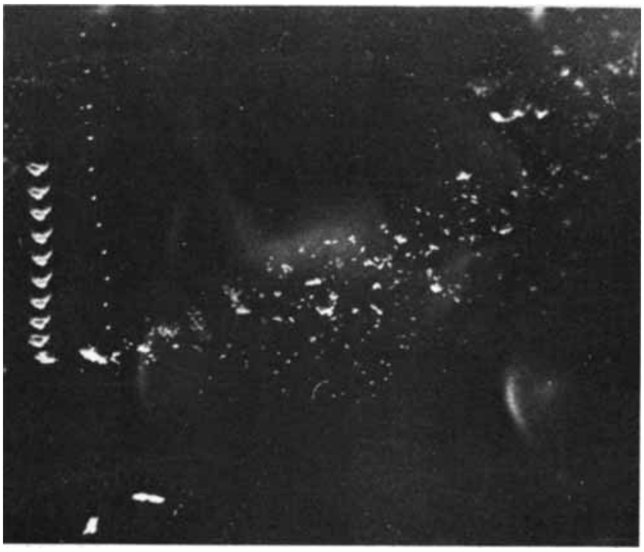

Figure 9. Breakage of kaolin-polyacrylamide floc at Re = 2646 and $x_1 \simeq 0.005$ m.

polymer aggregates were produced at pH-9.9, ionic strength $\simeq$ 0.003 m, and an initial dissipation rate of 90 cm^2/s^3; the coagulation was carried out in a baffled, agitated vessel of 13 liters capacity equipped with a side window and sample cell for instantaneous recording of the particle-size distribution. The results of the experiment indicate extremely rapid polymer adsorption, for within 60 seconds of polyacrylamide addition, flocs with mean diameters exceeding 1000×10^{-6} m were detected. At $t = 600$ s, ϵ was increased from 90 to 650 cm^2/s^3 to characterize the size of the stronger second-level structures, and the results are shown in Figure 10. We estimate that k_1 in Eq. 16 has an approximate value for this system on the order of 1×10^7 $cm^3/mol{\cdot}s$ and that $c_1 \simeq 4 \times 10^{-13}$ mol/cm^3. These are rough estimates. In Eq. 20, for the aggregation experiment performed, we find $\gamma \simeq 0.1$ s^{-1}, $b \simeq 1.89 \times 10^4$, and $k_3 \simeq 4.53 \times 10^{-5}$ cm/s, with the initial condition that at $t = 0$, $dn_2/dt \simeq 125$ $cm^{-3}{\cdot}s^{-1}$. The diametric range of second-level aggregates was arbitrarily assumed to extend from the threshold of detection by image analysis (approximately 10×10^{-6} m) to 450×10^{-6} m.

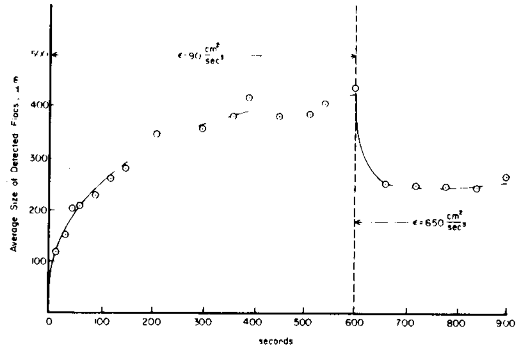

Figure 10. History of kaolin-polymer coagulation experiment illustrating the effect on an instantaneous increase in dissipation from 90 to 650 cm^2/s^3.

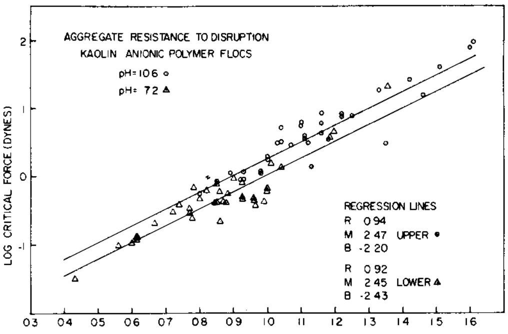

Figure 11. Experimental strength measurements for kaolin-polyacrylamide flocs.

The critical force requirement for kaolin-polymer flocs could also be correlated with parent aggregate size. At a pH of 10.6 with ionic strength similar to previous cases

$$B = 32.5d^{2.47} \tag{24}$$

with a correlation coefficient of 0.94. At pH = 7.2

$$B = 24.8d^{2.45} \tag{25}$$

with a correlation coefficient of 0.92. The data used to generate Eqs. 24 and 25 are shown in Figure 11. It is apparent from Eqs. 24 and 25 that the kaolin-polyacrylamide flocs are considerably stronger than their counterparts formed with $FeCl_3$ alone; under slightly alkaline conditions at $d = 0.15$ cm, they are more than twice as strong. Observation of the breakup of a few small flocs, which are very difficult to handle individually, indicates a decrease dependence of critical force upon parent aggregate diameter in both kaolin-Fe^{+III} and kaolin-polyacrylamide cases. The number tested, however, is too small to permit elaboration.

An important objective of work in this area is quantification of the breakage mode, or mean number of daughter particles produced upon fragmentation. It is clear that the breakage mode must depend upon both the mechanism and level of aggregation, and it certainly must be a function of local hydrodynamic parameters such as the dissipation rate, ϵ. Our present experimental technique does not permit accurate determination of ϵ although we have, by the inviscid estimate: $\epsilon \sim u^3/\ell$, obtained rough relative magnitudes of ϵ for the five examples of deaggregation shown in Figures 3 through 7. Using the mean x_1 – component of jet velocity and the jet width (both evaluated at the plane of initial jet-floc interaction) for u and ℓ, respectively, we find

$$\epsilon_9:\epsilon_7:\epsilon_4:\epsilon_5:\epsilon_8 = 80:65:10:4:1$$

While the correlation between breakage mode and dissipation rate is evident in the photographs, it cannot presently be quantified.

The model description of floc strength at the second level of aggregation indicates that the strength per unit mass should begin to decrease with increasing size at a diameter of about 150×10^{-6} to 200×10^{-6} m for kaolin-polyacrylamide structures formed under the conditions employed in this study. Indeed, the model indicates that floc strength for larger particles at the second level should be roughly proportional to $d^{-0.9}$. Experimental determination of kaolin-polyacrylamide floc strength suggests that strength per unit mass of large aggregates is approximately proportional to $d^{-1/2}$ We feel that the present theory is capable of little more than qualitatively describing the variation of strength with size because third-level structures, which are certainly less dependent upon polymer adsorption kinetics, begin to form very rapidly after the initiation of coagulation. Furthermore, the macromolecule undergoes a lengthy period of readjustment after arriving at the interface, as noted by Silberberg (1962), and this kinetic behavior is not considered by the model. It should be noted, however, that experiments of the type summarized by Figure 10 tend to substantiate the location of the maximum in the loc strength model; i.e., kaolin-polymer fragments with diameters $\lesssim 250 \times 10^{-6}$ m can withstand elevated dissipation rates for prolonged periods.

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NOTATION

a = constant
A = Hamaker constant
A_f = cross-sectional area at breakage plane
b = seed parameter for aggregation
B = binding force
c_o = initial polymer concentration
c_1 = constant
d = particle or floc diameter
e = electrostatic charge
F = force of interaction
G = mean velocity gradient
h = interparticle distance
k = Boltzmann constant
k_1 = adsorption rate constant
k_2 = deaggregation rate constant
k_3 = third-level formation rate constant
n_c = number of particle-particle contacts
n_i = number concentration of type i particles
n_1 = number or number concentration of primary particles
N_i = number concentration of type i ions
p = porosity of floc structure
r = particle or floc radius
r_1 = separation in x_1-direction
R_{11} = longitudinal correlation coefficient
t = time
u = characteristic eddy velocity
U = mean velocity
v_j = volume of j-level structure
V_2 = total aggregate volume at second level
x_1 = coordinate direction (with origin at jet orifice)
z_i = charge on ion i

Greek Letters

α = fractional volume of immobilized water
β = dielectric constant
γ = rate constant for orthokinetic flocculation
δ = thickness of ionic atmosphere
ϵ = dissipation per unit mass
η = collisional effectiveness factor
θ = fractional surface coverage
ρ = density
σ = floc strength constant
τ_y = floc yield stress
ψ = interaction potential
Ω = colloidal particle volume fraction

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